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*Note the following correction to Volume 28. Page 14, formula (4.12) through page 17, line 23 should be exchanged with page 70, line 8, through page 72, next-to-last line.*

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